Journal of Engineering Physics and Thermophysics, Vol. 75, No. 4, 2002

FINITE-ELEMENT AXIALLY SYMMETRIC MODEL OF THE THERMAL REGIME UNDER SELF-PROPAGATING HIGH-TEMPERATURE SYNTHESIS OF BLANKS IN A FREE-FLOWING SHELL

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UDC 621.1.016.4

By the finite-element method the problem of nonstationary heat conductivity in the system of three bodies of finite sizes with an internal moving source of the first kind (burning front) has been solved. The influence of various factors on the temperature field parameters under HTS pressing of alloys of the Ti–C–Ni system has been investigated. It has been found that in the synthesis products internal cooling conditions under which the contact surface temperature behind the burning front remains constant are realized and temperature equalization throughout the blank volume occurs. Under internal cooling conditions the material has the highest plasticity and compactability.

One method of obtaining refractory compounds and materials based on them is self-propagating high-temperature synthesis (HTS), itself representing a kind of burning. To obtain high-density materials, HTS products heated by the burning wave are subjected to pressing (HTS pressing technology). Unlike the hot pressing of inert powders at which, due to the external heating, isothermal conditions are provided, under pressing of HTS products their continuous cooling occurs. Therefore, to decrease the heat loss, one should provide heat insulation of synthesis products from the cold deforming tool. At HTS pressing this problem is solved by carrying out synthesis followed by deformation in a sand shell. The ability of synthesis products to be plastically deformed and compacted is primarily due to the temperature conditions of deformation. Therefore, the investigations of the thermal regime of the HTS pressing process are of scientific and practical importance.

The known works devoted to the thermal regime of the HTS process consider the axially symmetric problem of nonstationary heat exchange with movement of the burning front along the symmetry axis of a cylindrical system [1]. At HTS pressing a blank having the form of a round or quadratic plate is located in a cylindrical mould perpendicular to the symmetry axis. The burning reaction is initiated from the side surface or from the center of the charge blank and the burning front moves perpendicular to the symmetry axis. In [2], the thermal regime in the two-dimensional approximation has been investigated for the case of firing from the side surface of the blank. The present paper aims at mathematical modeling and obtaining of the laws of the formation of the thermal regime in the case of burning of exothermal mixtures at firing from the blank center. To model the process of heat exchange between the synthesis products and the environment, the finite-element method (FEM) is used.

The thermal regime parameters depend on the position of the point of initiation of the burning reaction determining the burning path length l_b and the burning time t_b of the charge blank. The cooling time of already synthesized volumes increases proportionally to the latter. To provide a high deformation temperature and plasticity of the material, it is desirable to have minimum values of l_b and t_b. Therefore, the firing at the center of blanks is optimum. It should be noted that in the square blank before the moment the burning wave goes to the side surface there is an axially symmetric and then a three-dimensional heat exchange. In the round plate, throughout the burning and on completion of the synthesis an axially symmetric heat exchange is realized. Below we consider the HTS pressing of round blanks and the axially symmetric nonstationary heat-conductivity problem, which is formulated as follows.

A round blank of thickness $2h_1$ and radius R_1 is placed in a sand shell and a steel cylindrical matrix. The sizes of the shell and the matrix are known. At the initial time, at the blank center a burning reaction with known

Samara State Technical University; email: mvm@sstu.edu.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 4, pp. 145–150, July–August, 2002. Original article submitted August 3, 2001; revision submitted January 24, 2002.

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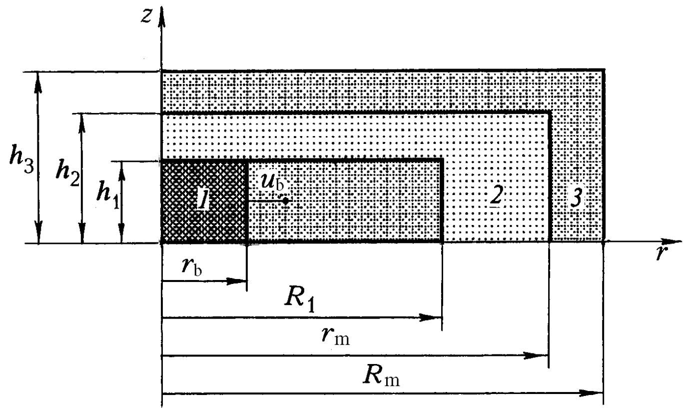


Fig. 1. Scheme of the object of modeling: 1) blank; 2) shell; 3) die.

burning temperature T_b and burning velocity u_b is initiated. Heat exchange between synthesis products and shell and shell and tool is realized under boundary conditions of the fourth kind with a perfect thermal contact. At the tool–environment boundary, boundary conditions of the third kind take place. We are to find the temperature field of the system of three contacting bodies at arbitrary time t. The design diagram is given in Fig. 1. In connection with the axial symmetry a fourth part of the meridional section in cylindrical coordinates r and z are considered. The burning front is assumed to be flat and moving in the direction of the r-axis.

The mathematical formulation of the axially symmetric heat-exchange problem at the burning stage includes:

1) the system of three differential equations of nonstationary heat conductivity in cylindrical coordinates

$$c_i \rho_i \frac{\partial T_i (r, z, t)}{\partial t} = \lambda_i \left(\frac{\partial^2 T_i (r, z, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T_i (r, z, t)}{\partial r} + \frac{\partial^2 T_i (r, z, t)}{\partial z^2} \right); \tag{1}$$

2) the boundary conditions: at the blank–shell and shell–tool boundaries the conditions are of the fourth kind

$$\lambda_1 \frac{\partial T_1 (r, h_1, t)}{\partial z} = \lambda_2 \frac{\partial T_2 (r, h_1, t)}{\partial z}, \quad T_1 (r, h_1, t) = T_2 (r, h_1, t),$$
$$\lambda_2 \frac{\partial T_2 (r, h_2, t)}{\partial z} = \lambda_3 \frac{\partial T_3 (r, h_2, t)}{\partial z}, \quad T_2 (r, h_2, t) = T_3 (r, h_2, t), \tag{2}$$

and at the tool–environment boundary ($z_3 = h_1 + h_2 + h_3$) the conditions are of the third kind

$$\lambda_3 \frac{\partial T_3 (r, z_3, t)}{\partial n} + \alpha \left[T_3 (r, z_3, t) - T_s \right] = 0; \tag{3}$$

3) the initial conditions:

$$T_1 (0, z_1, 0) = T_b, \quad T_2 (r, z, 0) = T_s, \quad T_3 (r, z, 0) = T_s; \tag{4}$$

4) the equation of motion of the burning front

$$r_b = u_b t; \tag{5}$$

5) the temperature of the moving boundary of the first kind (burning front)

$$T_1 (r_b, z_1, t) = T_b; \tag{6}$$

6) the adiabatic condition before the burning front

$$\left.\frac{\partial T_1\,(r, z_1, t)}{\partial r}\right|_{r=r_b+0} = 0\,; \tag{7}$$

7) the condition of temperature field symmetry about the z- and r-axes

$$\frac{\partial T_i\,(0, z, t)}{\partial r} = 0\,, \quad \frac{\partial T_i\,(r, 0, t)}{\partial z} = 0\,.$$

In calculating the temperature field after the whole volume of the charge is burnt, Eqs. (5)–(7) are eliminated from the system of equations (1)–(7) and boundary conditions of the fourth kind on the cylindrical surface of the blank at $r = R_1$

$$\lambda_1 \frac{\partial T_1\,(R_1, z, t)}{\partial z} = \lambda_2 \frac{\partial T_2\,(R_1, z, t)}{\partial z}\,, \quad T_1\,(R_1, z, t) = T_2\,(R_1, z, t)$$

are added.

The solution of Eqs. (1) with the boundary conditions (2), (3) and the initial conditions (4) is equivalent to the finding of the variation functional minimum

$$J = \sum_{i=1}^{3} \frac{1}{2} \int_{V_i} \lambda_i \left[r \left(\frac{\partial T_i}{\partial r} \right)^2 + r \left(\frac{\partial T_i}{\partial z} \right)^2 + 2c_i\, \rho_i\, \frac{\partial T_i}{\partial t}\, T_i \right] dV_i + \int_S \frac{\alpha}{2}\, (T_3 - T_s)^2\, dS\,. \tag{8}$$

The distinguishing feature of functional (8) is the fact that at the burning stage the volume v_1 of hot synthesis products with which heat exchange occurs is the function of time

$$V_1 = \pi r_b^2 h_1 = \pi h_1 u_b^2 t^2\,.$$

The sought temperature field was found by the FEM. Axially symmetric finite elements (FE) of the triangular section and the linear approximation of the temperature inside the element were used. When broken down into finite elements, the entire region is first covered with a rectangular mesh and then the obtained rectangles are divided by diagonals into two triangles. In the regions of the boundaries of contact blank–shell heat exchange with high temperature gradients, crowding of the FE mesh is fulfilled.

The stationary condition of functional (8) leads to the following discrete differential equation in matrix form:

$$[C]\,\frac{\partial \{T\}}{\partial t} + [\Lambda]\cdot\{T\} = \{\mathbf{F}\}\,. \tag{9}$$

The matrix differential equation (9) was solved by the finite-difference method according to the implicit-difference scheme

$$([C] + \Delta t\cdot[\Lambda])\cdot\{T_k\} = [C]\cdot\{T_{k-1}\} + \Delta t\cdot\{\mathbf{F}\} \tag{10}$$

The matrix elements $[C]$, $[\Lambda]$, and $\{\mathbf{F}\}$ are determined by the known dependences for the axially symmetric elements of the triangular section [3].

The technological time of temperature field formation t consists of the burning time t_b and the pressing delay time t_d: $t = t_b + t_d$. The value of the delay time t_d is made up of the operating time of the press (≈0.5 sec) and the time given by the researcher. Burning front motion was initiated by increasing stepwise the number of finite elements of the blank participating in the heat exchange. For the rectangular finite-element mesh the whole area represents a set

TABLE 1. Thermophysical Properties of Materials

Material	λ, W/(m·K)	c, J/(kg·K)	ρ, kg/m^3	T_b, ^oC	u_b, mm/sec
TiC–20% Ni	12.1	967.5	2700	2400	15
Sand [6]	0.326	795.0	1500	–	–
Steel tool [6]	32.0	561.0	7800	–	–

of vertical columns and horizontal layers. In the approximation of the flat burning front, the synthesized material volume increases, in one step, by the volume of the elements of one column of the blank. In the first step, only the elements of the first column of the blank of width Δr_1 participate in the heat exchange. In the second step, the burning front is displaced by the width of the elements of the second column of the blank Δr_2; in the third step — by the width of the elements of the third column of the blank Δr_3, and so on. In the nth step, the burning time t_{bn} of the new column of the blank of width Δr_n is

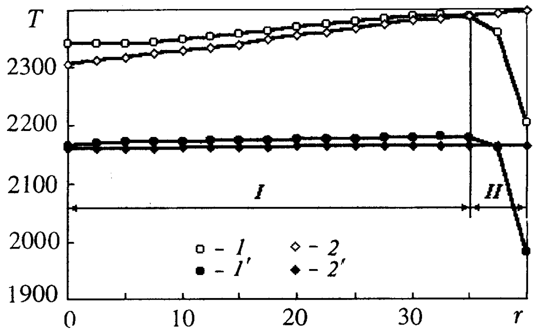
$$t_{bn} = \frac{\Delta r_n}{u_b} . \tag{11}$$

During this time a cooling of the blank also occurs. To determine the optimum steps by the time axis, we compared the solution of the one-dimensional equation (10) with the exact analytical solution of the one-dimensional problem on the cooling of an infinite layer of substance in an unbounded medium under boundary conditions of the fourth kind [4]. In analyzing the solutions, it has been found that the value of the time step t_{bn} calculated by (11) and the component for the real values of Δr_n and u_b of ~0.5–1 sec are a rather course discretization of the time axis. Therefore, the time t_{bn} was broken up into m intervals and Eq. (10) was solved with a step $\Delta t_n = t_{bn}/m$. It has been found that at $m = 5$ the numerical solution differs from the analytical one by no more than 1%.

At each nth step of burning, the unknown is the initial contact temperature T_{cont0} of the newly formed blank–shell contact surface. Therefore, we first calculated, by equations (10) at $\Delta t = 10^{-6}$ sec, the temperature T_{cont0} of the new contact node and then solved Eqs. (10) with a step Δt_n and determined the temperature fields at a current instant of time. To solve the system of linear equations (10), we used the Seidel iterative procedure with an accuracy of temperature calculation of 0.5^oC.

The laws of the temperature field formation were investigated under HTS pressing of the Ti–C–20% (mass) Ni system. Burning of this system is due to the highly exothermal reaction of titanium carbide formation: Ti + C = TiC; inert nickel serves as a binder. The thermophysical properties of the HTS products, sand, and steel die are given in Table 1. The data on the burning temperature t_b were taken from [5] and the value of the burning velocity u_b was determined experimentally. The burning temperature T_b for all compositions exceeds the eutectic temperature of the TiC–Ni system of 1280^oC, and the synthesis products consist of solid particles of titanium carbide and carbide-nickel melt. The thermophysical properties of the solid-liquid material were calculated by the dependences of [7] and, in accordance with the phase diagram, the quantity of the liquid and solid phases, the melt composition, and the material porosity equal to 50% were taken into account. The heat-transfer coefficient was taken as $\alpha = 44$ W/(m^2·K).

In the first stage, we investigated the thermal conditions for the synthesis of the type nomenclature of blanks and standard technological parameters of the process. We considered the synthesis of a round blank with a radius $R_1 = 40$ mm and thickness $h_{bl} = 2h_1 = 14$ mm in a matrix with $r_m = 55$ mm and $R_m = 62.5$ mm. The sand shell thickness was $h_{sh} = h_2 = 10$ mm; the thickness of the tool was $h_t = h_3 - h_2 = 15$ mm; the delay time of pressing was $t_d = 0.5$ sec. Figure 2 shows the temperature distribution over the blank radius in the case of synthesizing the TiC–20% Ni melt. Characteristic of the temperature field is the inhomogeneous temperature distribution in the bulk of the blank. This holds true for both the temperature of the contact surface of the blank T_{cont} and the temperature of its central part T_{cent}. The temperature field inhomogeneity is due to two factors. First, the blank has three contact heat-exchange boundaries that are heat sinks: two planes of support and a side cylindrical surface. In the vicinity of these boundaries the "coldest" zones with a high temperature gradient are formed. Second, when the blank is heated by a moving burning front the cooling time t_{cool} of individual zones depends on their position relative to the firing point:



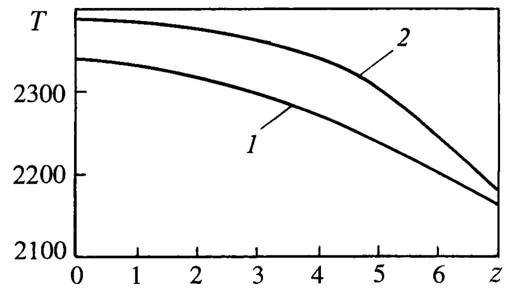


Fig. 2. Changes in the temperatures of the contact surface T_{cont} (1′, 2′) and the center T_{cent} (1, 2) along the blank radius: 1, 1′) solution of the two-dimensional problem by the FEM; 2, 2′) analytical solution of the one-dimensional problem. T, ^{o}C; r, mm.

Fig. 3. Temperature distribution with blank height: 1) $r = 0$ mm ($t_{cool} = 3.2$ sec); 2) $r = 35$ mm ($t_{cool} = 0.8$ sec). T, ^{o}C; z, mm.

$$t_{cool} = \frac{R_1 - r}{u_b} + t_{bl}\,, \quad 0 \le r \le R_1\,. \tag{12}$$

According to (12), as the firing point moves away ($r = 0$) and the r-coordinate increases, the cooling time decreases and the blank temperature increases.

On completion of the synthesis, in the blank two characteristic zones connected with the corresponding contact heat-exchange boundary are formed. In the central part of the blank, zone *I* is situated. This zone is characterized by a weak dependence of the contact temperature T_{cont} on the cooling time and the r-coordinate, throughout the zone $T_{cont} \sim$ const. As the cylindrical surface of the blank is approached ($r \to R_1$), zone *II*, in which the temperatures T_{cont} and T_{cent} sharply decrease, is formed.

Figure 3 shows the temperature distribution with blank height (z-coordinate) at the center of the blank ($r = 0$, $t_{cool} = 3.2$ sec) and at the boundary of zones *I* and *II* ($r = 35$ mm, $t_{cool} = 0.9$ sec). The small cooling time (curve 2) is characterized by a sharp difference in temperature between the central and contact volumes. With increasing cooling time at a practically constant temperature T_{cont} (for $z = 7$ mm) temperature equalizing in the blank thickness due to the cooling of the central volumes occurs, and in zone *I* the regime of internal cooling is thereby realized [4].

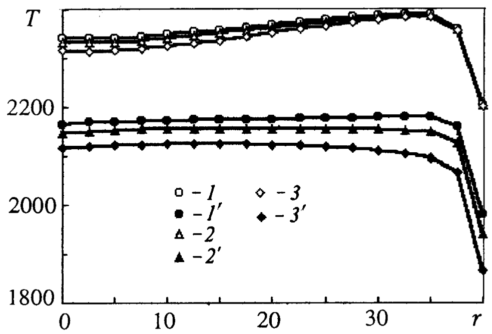
The results of the numerical solution were compared with the results of the analytical solution of the one-dimensional problem on the cooling of an infinite layer placed in an unbounded medium [4]. The cooling time of the infinite layer was taken to be equal to the cooling time of the section with a variable r-coordinate and was calculated by dependence (12). In the cooling of the infinite layer, contact heat exchange occurs only in one direction — along the z-coordinate. The stationary value of the temperature T_{cont} for the infinite layer corresponds to the temperature T_{cont0} that instantaneously settles at the boundary of the infinite layer cooled in the unbounded medium [4]:

$$T_{cont0} = T_s + (T_b - T_s)\,\frac{K_{\varepsilon 1}}{K_{\varepsilon 1} + K_{\varepsilon 2}}\,. \tag{13}$$

The results of the analytical solution and of the solution by the FEM (Fig. 2) are practically identical (the difference does not exceed 1%), and in the first approximation, to evaluate the temperature field in zone *I* with the regime of internal cooling, we can use the analytical solution of the one-dimensional problem.

Consider the influence of various factors on the thermal regime of HTS pressing of synthesis products of the Ti-C–20% Ni system.

The thickness of the shell between the blank and the tool is of paramount importance for the thermal regime. The sand shell and the tool in the aggregate represent a two-layer medium. The effective thermophysical properties of such a medium are determined by the sizes and individual properties of the components of the bodies. With decreasing



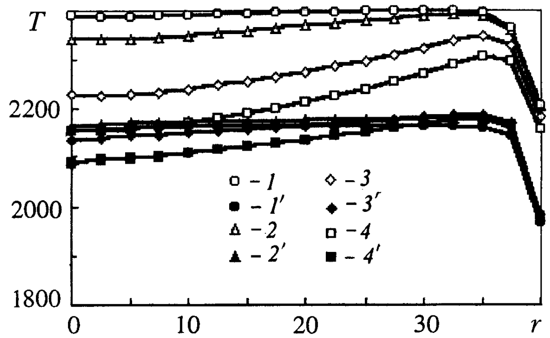


Fig. 4. Influence of the shell thickness h_{sh} on the distribution of temperatures T_{cent} (1, 2, 3) and T_{cont} (1′, 2′, 3′) over the blank radius at $h_{bl} = 14$ mm and $t_d = 0.5$ sec: 1 and 1′) $h_{bl} = 10$ mm; 2 and 2′) $h_{bl} = 5$ mm; 3 and 3′) $h_{bl} = 1.5$ mm. T, ^{o}C; r, mm.

Fig. 5. Influence of the blank thickness h_{bl} on the distribution of temperatures T_{cent} (1, 2, 3, 4) and T_{cont} (1′, 2′, 3′, 4′) over the blank radius at $h_{sh} = 10$ mm and $t_d = 0.5$ sec: 1 and 1′) $h_{bl} = 20$ mm; 2 and 2′) $h_{bl} = 14$ mm; 3 and 3′) $h_{bl} = 10$ mm; 4 and 4′) $h_{bl} = 6$ mm. T, ^{o}C; r, mm.

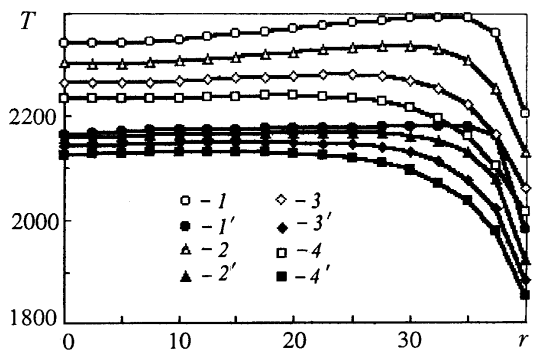


Fig. 6. Influence of the pressing delay time t_d on the distribution of temperatures T_{cent} (1, 2, 3, 4) and T_{cont} (1′, 2′, 3′, 4′) over the blank radius at $h_{bl} = 14$ mm and $h_{sh} = 10$ mm: 1 and 1′) $t_d = 0.5$ sec; 2 and 2′) $t_d = 2$ sec; 3 and 3′) $t_d = 4$ sec; 4 and 4′) $t_d = 6$ sec. T, ^{o}C; r, mm.

thickness of the sand shell the effective coefficient of thermal activity $K_{\varepsilon 2}$ increases and, according to (13), the initial temperature T_{cont0} and, therefore, the current temperature of the contact surface T_{cont} decrease. The results of the numerical experiment confirm this rule: as the shell thickness is decreased from $h_{sh} = 10$ mm to $h_{sh} = 1.5$ mm, the contact surface temperature T_{cont} monotonically decreases (Fig. 4). The temperature of the central part of the blank T_{cent} practically remains unchanged. As a result, the inhomogeneity of the temperature distribution with thickness of the blank increases and its mean temperature decreases.

The influence of the blank thickness $h_{bl} = 2h_1$ on the laws of change in temperatures T_{cont} and T_{cent} is given in Fig. 5. Variation in h_{bl} at a constant thickness of the shell h_{sh} produces no effect on the values of the thermal activity coefficients $K_{\varepsilon 1}$ and $K_{\varepsilon 2}$ and the initial contact temperature T_{cont0}. Accordingly, there is a weak dependence of the contact surface temperature T_{cont} on the blank thickness. The temperature at the blank center changes in the opposite manner. A decrease in the blank thickness and mass leads to a faster cooling of the central volumes of the blank and a decrease in its mean temperature. The regime of internal cooling at which $T_{cont} = \text{const}$ remains when h_{bl} decreases to 8 mm. Cooling of thin blanks with $h_{bl} < 8$ mm occurs at a high rate and the regime of internal cooling is not observed.

An important technological parameter is the pressing delay time t_d. As it increases, the cooling of the blank is accompanied by a decrease in the temperature gradients (Fig. 6). At small values of the delay time ($t_d \le 2$ sec) the temperature T_{cont} changes slightly and the regime of internal cooling at which the internal volumes of the blank markedly cool down with temperature T_{cent} is realized. The boundary of zones *I* and *II*, which was the hottest at the moment of complete burning of the blank, cools down most rapidly. As t_d increases due to the heat conductivity, the temperature T_{cent} becomes equal along the layer length and at $t_d \geq 6$ this high-temperature region disappears.

The ability of an inhomogeneously heated body for plastic deformation is determined by the temperature of the coldest zones. For HTS pressing this is the contact surface temperature T_{cont}. The maximum level of temperature T_{cont} throughout the blank surface corresponds to the regime of internal cooling. This state gives the top estimate of the deformation temperature of synthesis products. An increase in the dimensions of the sand shell leads to an increase in the absolute level of temperature T_{cont} and, in terms of plasticity and compactibility, produces a positive effect on the temperature regime of deformation. At the same time, with increasing h_{sh} the tool–shell contact rigidity decreases, which, together, with the temperature field inhomogeneity and the rheological properties, leads to a shape distortion and a low dimensional accuracy of the blank. Therefore, the question of optimum dimensions of the heat-insulating shell should be resolved by simultaneously investigating the thermal regime and the process of deformation of synthesis products in shells of various dimensions.

Thus, the finite-element model of heat exchange has made it possible to reveal the main laws of thermal regime formation under axially symmetric burning and cooling of round blanks of finite sizes in a heat-insulating sand shell. Characteristic of the temperature field in the synthesis products is a substantial inhomogeneity caused by the presence of contact heat-exchange boundaries and the nonisochronism of heating and cooling of the blank. Under nonstationary heat exchange in the blank–shell–tool system, the regime of internal cooling, at which the contact surface temperature behind the burning front remains constant and the temperature becomes equal throughout the bulk of the blank, is possible. The sizes of the region with the regime of internal cooling depend on the thermokinetic parameters of the HTS mixtures and the dimensions of the blank being synthesized and of the heat-insulating shell. In the regime of internal cooling, the material has the highest plasticity and compactibility. Accordingly, the optimum technological parameters of the process can be determined from the condition of the maximum volume of the blank with the regime of internal cooling.

NOTATION

T_i, temperature of bodies; c_i, ρ_i, and λ_i, specific heat, specific density, and heat conductivity coefficient of system bodies; h_i, characteristic sizes of system bodies; V_i, volume of system bodies ($i = 1, 2, 3$); t, time; r and z, cylindrical coordinates; α, heat-transfer coefficient; T_s, medium temperature; S, area of the tool with convective heat exchange; T_b, burning temperature; u_b, burning velocity; $[C]$, heat matrix; $[\Lambda]$, conduction matrix; $\{\mathbf{F}\}$, heat load vector; $\{T_{k-1}\}$ and $\{T_k\}$, matrices of nodal temperature values at the beginning and end of the time interval Δt; r_b, burning front radius, h_{bl}, h_{sh}, and h_t, thickness of the blank, shell, and tool; R_1, blank radius; r_m and R_m, inner and outer radii of the matrix; t_b, burning time; t_d, delay time of pressing; t_{cool}, cooling time; T_{cont}, contact temperature; T_{cont0}, initial contact temperature; T_{cent}, temperature at the blank center; $K_{\varepsilon 1} = \sqrt{\lambda_1 c_1 \rho_1}$ and $K_{\varepsilon 2} = \sqrt{\lambda_2 c_2 \rho_2}$, criteria of thermal activity of the blank and shell. Subscripts: sh, shell; b, burning; bl, blank; cont, contact; cent, center; s, medium; m, matrix; d, delay; t, tool; cool, cooling.

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